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THE COMPLEX RADIATION-RESISTANCE OF AN ANTENNA SYSTEM IN THE
PRESENCE OF ITS ELECTROMAGNETIC INTERACTION WITH ANOTHER ANTENNA SYSTEM *

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We consider two mutually-connected antenna systems each of which has a linear vibrator L and reflector S of arbitrary shape. The volume in which the considered antenna systems are disposed is so bounded by a closed surface S_0 that the surfaces of the reflectors belonging to the antenna systems coincides with part of the closed surface (see the figure). Let us find the complex radiation-resistance of antenna system 1 when it interacts electromagnetically with antenna 2.

In order to solve this problem we employ a lemma similar to Lorentz' lemma (1-4). When applied to two arbitrary electromagnetic fields $\vec{E}_1, \vec{H}_1$ and $\vec{E}_2, \vec{H}_2$ given within the closed surface S_0 and excited by the linear conductors L_1 and L_2 with points I_1 and I_2 of identical frequency ω (the conductors are located inside S_0), under the condition that certain arbitrary bodies which maintain their position without change are located inside this same volume, the lemma

*[Notes: Vectors are expressed thus: $\vec{E}$ etc.]

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assumes the following form:

$$\begin{aligned} \int_{S_0} (\vec{E}_1 \vec{H}_2^* + \vec{E}_2^* \vec{H}_1) d\vec{s} + \int_{\Sigma} (\vec{E}_1 \vec{H}_2^* + \vec{E}_2^* \vec{H}_1) d\vec{s} = \\ = -\frac{4\pi}{c} \left(\int_{L_1} \vec{E}_2 d\vec{l} + \int_{L_2} \vec{E}_1 d\vec{l} \right). \end{aligned} \quad (1)$$

Here Σ is the surface that excludes from the volume all those places where the continuity of the field is disrupted; $d\vec{s}$ is a vector coinciding in direction with the vector of the external normal to the corresponding elements of the surface. The asterisk sign *, as usual, designates the complex conjugate quantity. In the demonstration of the lemma, the conductivity of the medium found within S_0 is taken equal to zero.

In view of the fact that the demonstration of the lemma differs quite insignificantly from the demonstration of the ordinary lemma of Lorentz, we excuse ourselves from deriving it.

We shall understand by $\vec{E}_1$ and $\vec{H}_1$ the field which is excited by the dipole of the transmitting antenna whose current is I_1 ; and by $\vec{E}_2$ and $\vec{H}_2$ the arbitrary auxiliary field which is excited by the linear current I_2 within the closed surface S_0 . The receiving dipole we shall consider a certain foreign body which disrupts the regularity of the field where it is located, and in this connection we surround it with a small sphere of surface Σ . We designate the internal side of the metallic surface of the antennas by $S = S_1 + S_2$. In determining the auxiliary field $\vec{E}_2, \vec{H}_2$ we shall consider the surface to be ideally conducting. Then, neglecting the losses in the metal and setting $I_1 = I_2 = I$ and $L_1 = L_2 = L$, we obtain instead of equation (1) the following

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expression:

$$\int_{S_0-S} [\vec{E}_1 \vec{H}_2^*] d\vec{s} + \int_{\Sigma} ([\vec{E}_1 \vec{H}_2^*] + [\vec{E}_2^* \vec{H}_1]) d\vec{s} = -\frac{4\pi}{c} \left(\int_{\Sigma} I \vec{E}_2^* d\vec{l} + \int_{\Sigma} I^* \vec{E}_1 d\vec{l} \right). \quad (2)$$

Now let us determine the complex radiation-resistance (impedance) $\vec{Z}$ of the antenna, referring it to the current I_0 at a certain point of the conductor exciting the antenna. The well known formula for $\vec{Z}$ assumes the form:

$$\vec{Z} = -\frac{1}{2I_0 I_0^*} \int_{\Sigma} I^* \vec{E} d\vec{l}. \quad (3)$$

Employing equation (2) we can write formula (3) in the following form:

$$\vec{Z} = -\frac{c}{8\pi I_0 I_0^*} \int_{S_0-S} [\vec{E}_1 \vec{H}_2^*] d\vec{s} - \frac{1}{2I_0 I_0^*} \int_{\Sigma} I^* \vec{E}_2 d\vec{l} + \frac{c}{8\pi I_0 I_0^*} \int_{\Sigma} ([\vec{E}_1 \vec{H}_2^*] + [\vec{E}_2^* \vec{H}_1]) d\vec{s}. \quad (4)$$

Here the sign of the second integral is changed to the opposite sign and its complex-conjugate value is taken. Obviously these actions have not changed the values of the integral, since the integral is purely imaginary. Its real part, which expresses the active power expended in the maintenance of the auxiliary field $\vec{E}_2, \vec{H}_2$, under the assumptions made concerning the conductivity of the surface S_0 , equals zero.

Introducing the designation

$$\vec{Z}_0 = -\frac{1}{2I_0 I_2^*} \int_{\Sigma} I^* \vec{E}_2 d\vec{l}, \quad (5)$$

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we can reduce expression (4) to the following form:

$$\vec{Z} = \vec{Z}_0 + \frac{C}{8\pi I_0 I_0^*} \int_{S_0-S} \vec{E}_1 \vec{H}_2^* \cdot d\vec{s} + \frac{C}{8\pi I_0 I_0^*} \int_{\Sigma} (\vec{E}_1 \vec{H}_2^* \cdot \vec{n} + \vec{E}_2^* \vec{H}_1 \cdot \vec{n}) d\vec{s} \quad (6)$$

or, in practical units (watts, volts, and oersteds), to the form:

$$\vec{Z} = \vec{Z}_0 + \frac{1}{0.8\pi I_0 I_0^*} \int_{S_0-S} \vec{E}_1 \vec{H}_2^* \cdot d\vec{s} + \frac{C}{0.8\pi I_0 I_0^*} \int_{\Sigma} (\vec{E}_1 \vec{H}_2^* \cdot \vec{n} + \vec{E}_2^* \vec{H}_1 \cdot \vec{n}) d\vec{s} \text{ ohms} \quad (7)$$

As is apparent from equations (6) and (7), the complex radiation-resistance of the system considered consists of the sum of three terms. The first of these equals the radiation-resistance of the linear conductor L exciting the closed metallic surface; the second term represents that part of the radiation resistance which arises during the presence, in this surface, of an "aperture" with the dimension $(S_0 - S)$; and, finally, the third term expresses the influence of the receiver vibrator (oscillator).

In the case where receiver vibrator is a point dipole, the integrals over the surface of the sphere in expressions (6) and (7) can easily be calculated with the aid of the limit transition to a sphere of infinitely small radius. In this case it is necessary to keep in mind the character of the field's singularity at the point where the dipole is located.

Expressions (6) and (7) permit one to determine the active resistance of an antenna system in the presence of its interaction with another antenna system. Taking into account that the first term of the expression for $\vec{Z}$ is purely imaginary, we obtain immediately from equation (7) the following:

$$R = \frac{C}{0.8\pi I_0 I_0^*} \operatorname{Re} \int_{S_0-S} \vec{E}_1 \vec{H}_2^* \cdot d\vec{s} + \frac{C}{0.8\pi I_0 I_0^*} \operatorname{Re} \int_{\Sigma} (\vec{E}_1 \vec{H}_2^* \cdot \vec{n} + \vec{E}_2^* \vec{H}_1 \cdot \vec{n}) d\vec{s} \text{ ohms} \quad (8)$$

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Multiplying the preceding expression by $I_0 I_0^*$, we find the formula for the radiation power P to be:

$$P = \frac{C}{0.8} \operatorname{Re} \int_{S_0-S} \vec{E}_1 \vec{H}_2^* d\vec{s} + \frac{C}{0.8} \operatorname{Re} \int_S (\vec{E}_1 \vec{H}_2^* + \vec{E}_2^* \vec{H}_1) d\vec{s} \text{ watts} \quad (9)$$

The second term of this expression determines the power transmitted to the receiver antenna, and the first term determines the power lost through the "aperture" (S_0-S). Formulas (7), (8), (9), which determine the radiation resistance and radiation power, are convenient in that in order to use these formulas it is sufficient to know the field $\vec{E}_1, \vec{H}_1$ of the transmitting antenna and the field $\vec{E}_2, \vec{H}_2$ of the auxiliary source excited within the closed metallic surface.

The procedure for computing $\vec{E}_1$ and $\vec{H}_1$ is expounded in a previous work (5) of the author. As for the procedure for determining the auxiliary field $\vec{E}_2$ and $\vec{H}_2$ within the closed ideally conducting surface, it is well known from the proper literature.

We note that formulas (7), (8), (9) are correct for any choice of the geometrical surface S_0-S , which makes the surface of the antenna reflectors a closed surface.

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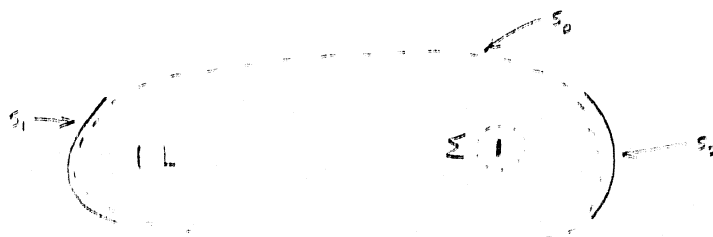


Figure of two antenna systems.

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